

Kaluza-Klein Cosmology With Modified Holographic Dark Energy

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Abstract

We investigate the compact Kaluza-Klein cosmology in which modified holographic dark energy is interacting with dark matter. Using this scenario, we evaluate equation of state parameter as well as equation of evolution of the modified holographic dark energy. Further, it is shown that the generalized second law of thermodynamics holds without any constraint.

Keywords: Kaluza-Klein cosmology; Modified holographic dark energy; Generalized second law of thermodynamics.

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1 Introduction

Recent cosmological observations [1]-[3] indicate that the universe is spatially flat and has an accelerated expansion. These observations lead to a matter called dark energy (DE) which has large negative pressure. The DE can be explained in terms of cosmological constant, acts like a perfect fluid with an equation of state, satisfying the observational data so far. However, this involves the problems of fine tuning and cosmic coincidence. Many dynamical

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models like phantom [4], quintessence [5], quintom [6], tachyon [7], generalized Chaplyng gas [8] etc. have been proposed to alleviate these problems. The nature of DE is still unknown in spite of many efforts for its investigation.

An alternative way to understand the early time inflation and late time acceleration of the universe is to modify gravity theories such as $f(R)$ theory, $f(T)$ theory, $f(G)$ theory and extra dimensional theories. Recently, the theory of extra dimensions has attracted many people. The world may have five dimensions is due to the idea of Kaluza [9] and Klein [10]. They used one extra dimension to unify gravity and electromagnetism and obtained a 5D general relativity. This idea has been used by many people for studying the models of cosmology and particle physics [11]-[13] (see review of the KK and higher dimensional unified theories [14]).

In cosmology of 5D with pure geometry in non-compact KK theory, one does not need to insert matter by hand because the matter is induced in 4D by 5D vacuum theory [15]. Actually, the curvature of 5D spacetime induces effective properties of matter in 4D. This is a consequence of the Campbells theorem [16]-[17] which states that any analytic N -dimensional Riemannian manifold can be locally embedded in $N + 1$ -dimensional Ricci flat Riemannian manifold. In this theory, the energy density of scalar field contributes to define the early inflation and late time acceleration which provides extra motivation in solving these problems [18]. People [19]-[21] explored this 5D theory by inserting matter instead of pure 5D geometry.

The nature of DE can also be studied according to some basic quantum gravitational principles, for example, holographic DE principle. According to this principle [22], the degrees of freedom in a bounded system should be finite and do not scale by its volume but with its boundary area. Cohen et al. [23] found that for a system with infrared (long distance) cutoff scale L and ultraviolet (short distance) cutoff scale Λ without decaying into a black hole, the quantum vacuum energy should be less than or equal to the mass of a black hole, i.e., $L^3 \rho_\Lambda \leq LM_p^2$. Here ρ_Λ is the vacuum energy density and $M_p = (8\pi G)^{-\frac{1}{2}}$ is the reduced Plank mass. Using this idea in cosmology, one can take L which satisfies this inequality with ρ_Λ as DE density.

There exist many cosmological versions of holographic principle in literature [24]-[29]. It is found [29] that this principle can be replaced by the generalized second law of thermodynamics (GSLT) for time dependent backgrounds. This is similar to the cosmological holographic principle given by Fischler and Susskind [24] for an isotropic open and flat universe with fixed

equation of state. It is found that cold dark matter (CDM) is decaying into DE [30, 31] which favors the interaction between these two components. Many models with interacting DE have been investigated [32]-[33]. In a recent paper [34], we have investigated the validity of GSLT for Bianchi type *I* model in which anisotropic dark energy is interacting with dark matter and anisotropic radiation. The holographic DE in extra dimension can be studied with the help of the mass of black hole in $N + 1$ dimensional spacetime [35] and modified holographic dark energy (MHDE) [36]. Liu et al. [38] has obtained some interesting results with MHDE in Dvali-Gabadaze Porrati brane world.

In this paper, we study equations of state parameter as well as evolution of MHDE in compact KK theory such that MHDE is interacting with DM to explore the dynamics of vacuum energy. Also, we investigate the GSLT in this scenario. The format of the the paper is as follows: In next section, the equation of state parameter and equation of evolution are formulated. Section **3** discusses the validity of GSLT. Finally, we give some concluding remarks in the last section.

2 Equations of State Parameter and Evolution

Here we take the modified holographic dark energy interacting with dark matter in compact Kaluza-Klein cosmology. We explore equation of state parameter as well as equation of evolution in this scenario. The metric representation of the KK universe [39] is given by

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2 \right], \quad (2.1)$$

where $a(t)$ is the scale factor, $k = -1, 0, 1$ is the curvature parameter for the closed, flat and open universe respectively. Suppose that the KK universe is filled with perfect fluid defined by the following energy-momentum tensor

$$T_{\mu\nu} = (P + \rho)U_\mu U_\nu - g_{\mu\nu}P, \quad (\mu, \nu = 0, 1, 2, 3, 4), \quad (2.2)$$

where $P = P_\Lambda + P_m$, $\rho = \rho_\Lambda + \rho_m$ are the pressure and density respectively. The subscripts Λ and m denote MHDE and DM respectively. Here U_μ is the

five velocity such that $U^\mu U_\mu = 1$. The Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \quad (2.3)$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R , $T_{\mu\nu}$ and κ are the Ricci tensor, the metric tensor, the Ricci scalar, the energy-momentum tensor and the coupling constant respectively. For the sake of simplicity, we take $\kappa = 1$. Using Eqs.(2.1) and (2.2) in Eq.(2.3), it follows that

$$\rho = 6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2}, \quad (2.4)$$

$$P = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2}. \quad (2.5)$$

For the flat universe $k = 0$, we have

$$\rho = 6\frac{\dot{a}^2}{a^2} = 6H^2, \quad (2.6)$$

$$P = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2}, \quad (2.7)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. The continuity equation, $T^{\mu\nu}_{;\nu} = 0$, gives

$$\dot{\rho} + 4H(\rho + P) = 0.$$

Now we assume that MHDE is interacting with the DM so that the continuity equations for DM and MHDE, respectively, take the form

$$\dot{\rho}_m + 4H(\rho_m + P_m) = Q_4, \quad \dot{\rho}_\Lambda + 4H(\rho_\Lambda + P_\Lambda) = -Q_4. \quad (2.8)$$

Here Q_4 is a new form of interacting term defined as [40]

$$Q_4 = 3b(\rho_\Lambda - \rho_m), \quad (2.9)$$

where b is a coupling constant. In a recent paper [41], Cai and Su found that Q (an interacting term) may cross the non-interacting line ($Q = 0$), i.e., the sign of Q changes around the red-shift variable $z = 0.5$. This leads to a big challenge for the interacting models as the general interaction terms cannot change the signs. However, this interaction term not only solves this problem but also consistent with GSLT using the arguments [40] both for the early

time when $T_m > T_d$ and for the late time when $T_d > T_m$ (T_m and T_d indicate temperature or dark matter and dark energy respectively).

In $N + 1$ dimensional spacetime, the mass of the Schwarzschild black hole is given by [35]

$$M = \frac{(N - 1)A_{N-1}r_H^{N-2}}{16\pi G},$$

where A_{N-1} denotes the area of unit N -sphere, r_H is the horizon scale of black hole, G is $N + 1$ dimensional gravitational constant which is related by $N + 1$ dimensional Plank mass M_{N+1} and the usual Plank mass in 4-dimensional spacetime. Also, $8\pi G = M_{N+1}^{-(N-1)} = \frac{V_{N-3}}{M_p^2}$, V_{N-3} is the volume of extra dimensional space. Thus

$$M = \frac{(N - 1)A_{N-1}r_H^{N-2}M_p^2}{2V_{N-3}}.$$

It is given that [36]

$$L^3 \rho_\Lambda \sim \frac{(N - 1)A_{N-1}L^{N-2}M_p^2}{2V_{N-3}}$$

implying that

$$\rho_\Lambda = \frac{c^2(N - 1)A_{N-1}L^{N-5}M_p^2}{2V_{N-3}},$$

where c is a constant parameter. For KK cosmology, i.e., for $N = 4$, it follows that

$$\rho_\Lambda = \frac{3c^2 A_3 L^{-1}}{2}. \quad (2.10)$$

Inserting the value of area of 4-sphere, we have

$$\rho_\Lambda = 3c^2 \pi^2 L^2. \quad (2.11)$$

The dynamical apparent horizon (a marginally trapped surface with zero expansion) is defined as [37]

$$h^{ab}\partial_a \tilde{r} \partial_b \tilde{r} = 0, \quad (a, b = 0, 1). \quad (2.12)$$

Using the FRW model, it has been proved that the radius of apparent horizon is given as

$$r_a = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}, \quad (2.13)$$

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$ and $h_{ab} = \text{diag}(-1, \frac{a^2}{\sqrt{1-kr^2}})$. It has also been proved that the apparent horizon coincides with the Hubble horizon $r_H = \frac{1}{H}$ for the flat FRW universe. In this paper, we take KK cosmology containing FRW as subspace with compact fifth dimension. This model has the same treatment of apparent horizon as in the case of flat FRW model [37]. Thus we can write

$$r_a = \frac{1}{H} = r_H = L. \quad (2.14)$$

Therefore, the infrared cutoff of the universe L in this KK flat universe is equal to the apparent horizon which coincides with Hubble horizon.

The equations of state are

$$P_\Lambda = \omega_\Lambda \rho_\Lambda, \quad P_m = \omega_m \rho_m. \quad (2.15)$$

The continuity equations (2.8) in effective theory are

$$\dot{\rho}_m + 4H(1 + \omega_m^{eff})\rho_m = 0, \quad (2.16)$$

$$\dot{\rho}_\Lambda + 4H(1 + \omega_\Lambda^{eff})\rho_\Lambda = 0, \quad (2.17)$$

where

$$\omega_m^{eff} = \omega_m - \frac{Q_4}{4H\rho_m}, \quad \omega_\Lambda^{eff} = \omega_\Lambda + \frac{Q_4}{4H\rho_\Lambda}. \quad (2.18)$$

Using Eqs.(2.11), (2.14) and (2.17), we have

$$\omega_\Lambda^{eff} = -1 - \frac{\dot{\rho}_\Lambda}{4\rho_\Lambda} = -1 - \frac{\dot{H}}{2H^2}. \quad (2.19)$$

It follows from Eq.(2.18) that

$$\omega_\Lambda = \omega_\Lambda^{eff} - \frac{Q_4}{4H\rho_\Lambda}$$

Inserting the values of Q_4 and ω_Λ^{eff} from Eqs.(2.9) and (2.19) respectively, we obtain

$$\omega_\Lambda = -1 - \frac{\dot{H}}{2H^2} - \frac{3b(\rho_\Lambda - \rho_m)}{4\rho_\Lambda}. \quad (2.20)$$

Equations (2.6) and (2.7) yield

$$\frac{\dot{H}}{2H^2} = -\frac{1}{2}\left(1 - \frac{\ddot{a}a}{\dot{a}^2}\right).$$

Substituting this value of $\frac{\dot{H}}{2H^2}$ in Eq.(2.20), we have

$$\omega_\Lambda = -\frac{1}{2} + \frac{\ddot{a}a}{2\dot{a}^2} - \frac{3b(\Omega_\Lambda - \Omega_m)}{4\Omega_\Lambda}, \quad (2.21)$$

where $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$, $\rho_c = 6H^2$. This is the equation of state parameter for the modified holographic dark energy.

The energy density of MHDE is

$$\rho_\Lambda = 3c^2\pi^2 L^2 = \frac{3c^2\pi^2}{H^2}$$

which leads to

$$\Omega_\Lambda = \frac{c^2\pi^2}{2H^4},$$

and hence

$$\dot{\Omega}_\Lambda = -2c^2\pi^2 \frac{\dot{H}}{H^5}.$$

Replacing $\frac{\dot{H}}{H^5}$ and simplifying, it follows that

$$\dot{\Omega}_\Lambda = 2c^2\pi^2 \left(\frac{\dot{a}^3}{a^3} - \frac{a^4\ddot{a}}{\dot{a}^5} \right). \quad (2.22)$$

Now

$$\frac{d\Omega_\Lambda}{da} = \frac{1}{Ha} \frac{d\Omega_\Lambda}{dt} = 2c^2\pi^2 \left[\frac{\dot{a}^2}{a^3} - \frac{\ddot{a}a^4}{\dot{a}^6} \right]. \quad (2.23)$$

This is the equation of evolution of the modified holographic dark energy.

3 Generalized Second Law of Thermodynamics

The discovery of black hole thermodynamics has led to the thermodynamics of cosmological models. Bekenstein [42] proved that there is a relation between an event horizon and thermodynamics of black hole. The event horizon of black hole is in fact the measure of entropy which is generalized to the cosmological models so that each horizon corresponds to an entropy. The generalized second law of thermodynamics is generalized in such a way that the sum of time derivative of each entropy must be increasing. Since horizon

is a function of time, so the change in horizon causes change in volume with respect to time. Consequently, energy and entropy also change and hence both states have the common source $T_{\mu\nu}$. We assume that temperature and pressure (which are the fundamental ingredients for the discussion of GSLT) remain the same.

Here, we explore the validity of the generalized second law of thermodynamics in the KK universe in which MHDE is interacting with DM. The first law of thermodynamics is

$$dS = \frac{PdV + dE}{T},$$

where T , S , E and P are temperature, entropy, internal energy and pressure of the system respectively. The corresponding entropies for MHDE and DM will become

$$dS_\Lambda = \frac{P_\Lambda dV + dE_\Lambda}{T}, \quad dS_m = \frac{P_m dV + dE_m}{T}. \quad (3.24)$$

We discuss the GSLT when the the system is in equilibrium. In this case, the temperature of the fluid and horizon are the same. We avoid the concepts like chemical potential. The temperature and entropy of horizon are defined as [37]

$$T = \frac{1}{2\pi r_a}, \quad S_h = \frac{A}{4G}.$$

In this FRW type KK cosmology, we have

$$T = \frac{1}{2\pi r_A} = \frac{1}{2\pi L}, \quad (3.25)$$

which corresponds to temperature of de-Sitter horizon and apparent horizon in flat FRW cosmology [37]. Thus, this definition of temperature and entropy of horizon work well in the FRW type KK universe. From Eq.(2.6), we have

$$L^{-2} = H^2 = \frac{1}{6}(\rho_\Lambda + \rho_m). \quad (3.26)$$

The volume of the system is

$$V = \frac{\pi^2 L^4}{2}. \quad (3.27)$$

Thermodynamical quantities are related to the cosmological quantities by the following relations

$$P_\Lambda = \omega_\Lambda^{eff} \rho_\Lambda, \quad P_m = \omega_m^{eff} \rho_m, \quad E_\Lambda = \frac{\pi^2 L^4 \rho_\Lambda}{2}, \quad E_m = \frac{\pi^2 L^4 \rho_m}{2}. \quad (3.28)$$

In four dimension, $S_h = \frac{2\pi^2 L^3}{4G} = 4\pi^3 L^3$ as $8\pi G = 1$ leads to

$$\dot{S}_h = 12\pi^3 L^2 \dot{L}. \quad (3.29)$$

Also, the time derivative of Eq.(3.24) yields

$$\dot{S}_\Lambda = \frac{P_\Lambda \dot{V} + \dot{E}_\Lambda}{T}, \quad \dot{S}_m = \frac{P_m \dot{V} + \dot{E}_m}{T}. \quad (3.30)$$

From Eqs.(3.25), (3.27)-(3.30), we get

$$\dot{S}_{total} = 4\pi^3 L^4 [(1 + \omega_\Lambda^{eff})\rho_\Lambda + (1 + \omega_m^{eff})\rho_m](\dot{L} - LH) + 12\pi^3 L^2 \dot{L}. \quad (3.31)$$

where S_{total} is the sum of three entropies. It follows from Eqs.(2.18)-(2.19) that

$$(1 + \omega_\Lambda^{eff})\rho_\Lambda + (1 + \omega_m^{eff})\rho_m = (1 + \omega_\Lambda)\rho_\Lambda + (1 + \omega_m)\rho_m. \quad (3.32)$$

Using Eqs.(2.16), (2.17) and (3.26), it turns out that

$$\dot{L} = \frac{L^3 H}{3} [(1 + \omega_\Lambda^{eff})\rho_\Lambda + (1 + \omega_m^{eff})\rho_m].$$

Substituting this value of \dot{L} in Eq.(3.31), we have

$$\dot{S}_{total} = \frac{4\pi^3 L^6}{3} [(1 + \omega_\Lambda)\rho_\Lambda + (1 + \omega_m)\rho_m]^2. \quad (3.33)$$

This implies that the time derivative of normal entropy plus horizon entropy is an increasing function. Hence the second law of thermodynamics holds for all time.

4 Discussion and Concluding Remarks

In this paper, we have taken compact FRW type KK universe along with MHDE interacting with DM. We have developed equation of state parameter as well as equation of evolution for this scenario. We conclude from the equation of evolution that its evolution depends on the scale factor and its derivatives. The first two terms in Eq.(2.21), i.e., $-\frac{1}{2} + \frac{\ddot{a}a}{2\dot{a}^2}$, can be written in terms of deceleration parameter q as $\frac{-(1+q)}{2}$. This indicates that the equation of state parameter of MHDE is an increasing function. Since the universe has an accelerating expansion with $q \leq -1$, this means that $q + 1 \leq 0$ implying $\frac{-(1+q)}{2} \geq 0$. Similarly, consider the third term in Eq.(2.21) involving the energy densities. The coincidence problem says that matter density is nearly equal to the DE density in the present era. It follows that $(\Omega_\Lambda - \Omega_m) \sim 0$ which implies that $\frac{3b(\Omega_\Lambda - \Omega_m)}{4\Omega_\Lambda} \sim 0$. This means that equation of state parameter depends majorally on the first two terms, hence it is an increasing.

Further, we have investigated that GSLT holds for all time and its validity is independent of the interacting form, equations of state parameters, fifth dimension and the geometry of the black hole. This validity of GSLT is proved for all times with the same results as in FRW universe [43]. Finally, we would like to mention here that GSLT also holds for non-compact KK type universe in which the variation along fifth dimension is very small.

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